

# BURSTING OF NEURONS UNDER SLOW WAVE STIMULATION

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**Abstract**—Hopf bifurcation of the H-H neuron was computed by numerical methods and discharge of H-H neuron under slow wave stimulation was studied by simulation. It is shown that slow wave can result in neuron bursting, especially the long time-course bursting, and onset of the bursting is related with Hopf bifurcation. The results prompt that bursting and supernormal excitability of neurons can be induced by slow excitatory synaptic reaction when it is big enough. This phenomenon probably can be used to explain the seizure of some diseases (such as epilepsy).

**Keywords**—H-H neuron, Hopf bifurcation, Bursting, Slow wave stimulation

## I. INTRODUCTION

For a long period of time, only the stimulation with enough steep gradient was thought of as the scheme that can fire neuron. And slow time-course reaction in synapse was only thought of as a modulation to fast synaptic efficacy. At the same time, pulse signals were used in the most medical experiments and model simulations. All along, there is little study of slow wave stimulation on neurons. Actually, slow excitatory synaptic reaction, such as slow excitatory postsynaptic current and potential, often appears in central and peripheral nervous system which active slowly and having time-course of seconds or minutes. Many diseases are related with slow synaptic reaction<sup>[1]</sup>. So, we must pay more attention to the effects of slow stimulation or slow synaptic reaction on neurons.

Bifurcations play important roles in analysis of dynamical systems, which express qualitative changes in behavior. A typical example of bifurcation in neuron dynamics is the transition from rest to periodic spiking activity. Hopf bifurcation is one of the most important bifurcation observed in neuron dynamics, since it describes the onset (or disappearance) of periodic activity, which is ubiquitous in the brain. A number of studies have been done on the bifurcation in the H-H neuron. Fukai et al<sup>[2]</sup> examined a global bifurcation structure of the H-H in the multiple-parameter space.

Our studies focus on the discharge of neurons under slow wave stimulation through bifurcation analysis and model simulation.

## II. METHOD

Hodgkin-Huxley neuron<sup>[3]</sup> (H-H neuron) is selected as the model in our studies. Hopf bifurcation of H-H neuron is computed by numerical methods<sup>[2]</sup>. Discharge of H-H neuron under slow wave stimulation is simulated with MATLAB SIMULINK. Two types of slow wave stimulation are used.

They are a ramp signal with slope of  $0.02\mu\text{A}/\text{cm}^2/\text{ms}$  and a half-sine-wave with frequency of  $0.05\text{Hz}$  and amplitude of  $30\mu\text{A}/\text{cm}^2$ .

The parameters of H-H neuron are set to the original values used by Hodgkin and Huxley<sup>[3]</sup>, only the temperature  $T$  as a variable, and the current  $I$  as the bifurcation parameter representing either an external stimulation current or a synaptic current.

## III RESULTS

### A. Bifurcation diagram while $T=18.5^\circ\text{C}$

One-parameter bifurcation diagrams (1BDs) was used, which give the relations between the equilibrium potentials and the maximal/ minimal values of the membrane potentials for periodic solutions as a function of  $I$ .

When  $T=18.5^\circ\text{C}$ , the 1BDs is shown in Fig.1. Nomenclature: *uH*-subcritical Hopf, *sH*-supercritical Hopf, *dc*-double cycle.  $I_{uH}=18.5673\mu\text{A}/\text{cm}^2$ ,  $I_{sH}=151.5824\mu\text{A}/\text{cm}^2$ ,  $I_{dc}=8.035\mu\text{A}/\text{cm}^2$

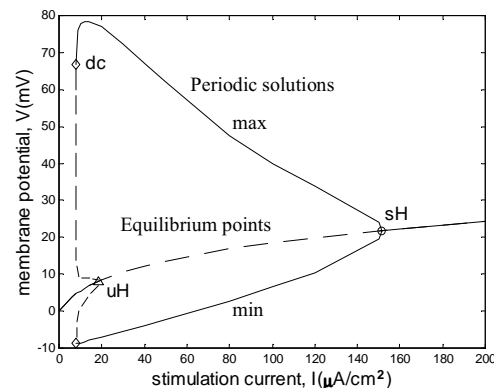


Fig.1. Bifurcation diagrams of an H-H neuron as a function of  $I$ .  $T=18.5^\circ\text{C}$ . Here  $I$  is the bifurcation parameter and  $V$  is the membrane potential of the limit states. Nomenclature: *uH*-subcritical Hopf, *sH*-supercritical Hopf, *dc*-double cycle. Solid lines represent stable periodic solutions and equilibrium points. Dashed lines represent unstable periodic solutions and equilibrium points.

From Fig.1, it can be seen clearly that the onset (or disappearance) of periodic activity depends on the intensity of the stimulation current  $I$ .

### B. Simulation of discharge while $T=18.5^\circ\text{C}$

Bursting of an H-H neuron under slow wave stimulation is shown in Fig.2.  $T=18.5^\circ\text{C}$ . (a) Ramp signal with slope of  $0.02\mu\text{A}/\text{cm}^2/\text{ms}$ . (b) Half-sine-wave with frequency of  $0.05\text{Hz}$  and amplitude of  $30\mu\text{A}/\text{cm}^2$ .

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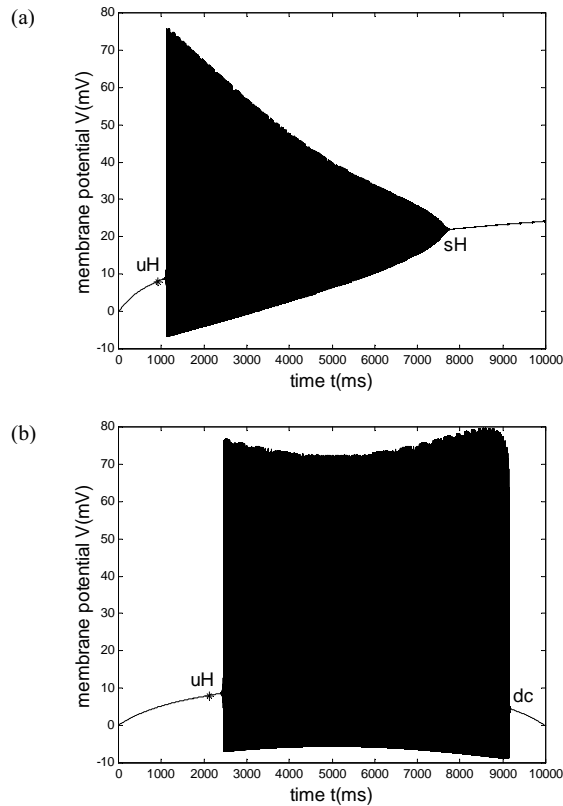


Fig.2. Bursting of an H-H neuron under slow wave current stimulation.  $T=18.5^{\circ}\text{C}$ . (a) Ramp current stimulation (slope is  $0.02\mu\text{A}/\text{cm}^2/\text{ms}$ ). (b) Half-sine-wave stimulation (frequency is  $0.05\text{Hz}$ , amplitude is  $30\mu\text{A}/\text{cm}^2$ ). Nomenclature:  $uH$ -subcritical Hopf,  $sH$ -supercritical Hopf,  $dc$ -double cycle.

In Fig.2, bursting is induced by both two types of slow wave stimulation. The onset of bursting is delayed significantly when the variable  $I$  passed slowly a bifurcation value, which is known as the slow passage effect<sup>[4]</sup>. So, under slow wave stimulation, bursting starts at  $uH$  point and ends at  $sH$  point or  $dc$  point.

### C. Bifurcation diagrams at different temperatures

Bifurcation diagrams at different temperatures are shown in Fig.3.

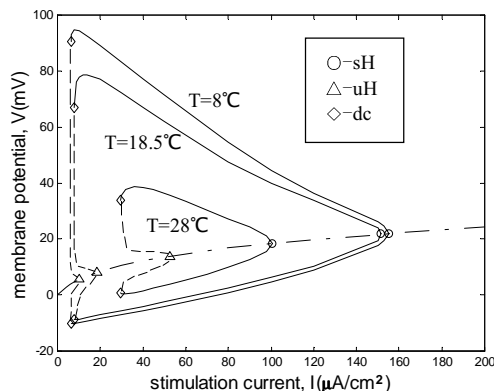


Fig.3. Bifurcation diagrams of the H-H neuron at different temperatures. Nomenclature:  $uH$ -subcritical Hopf,  $sH$ -supercritical Hopf,  $dc$ -double cycle.

The higher the temperature is, the narrower the bifurcation parameter range is. At the human body temperature, there is no Hopf bifurcation, and the bursting can not be induced any more by slow wave stimulation.

### IV. DISCUSSION AND CONCLUSION

(1) Bursting, especially the long time-course bursting, can be induced by slow wave stimulation. The onset of bursting induced by slow stimulation is related with Hopf bifurcation point. This is a complementarity to traditional concept of neuron's accommodation. Traditionally, it was well known that the stimulation must be of not only enough intensity but also enough steep to activate an action potential.

(2) This is a new understand to the effect of slow synaptic reaction to a neuron. It is shown that slow excitatory synaptic reaction can induce a neuron bursting by itself, not simply as an modulation to fast synaptic efficacy, if only it reaches enough intensity.

(3) It implies that long time-course bursting induced by slow excitatory synaptic reaction maybe just the seizure reason of some diseases (such as epilepsy). The slow passage effect can be shortened significantly by noise or weak input from other bursters, which provides a powerful mechanism for instantaneous synchronization of bursters.

(4) At the human body temperature, there is no Hopf bifurcation, and the bursting can not be induced any more by slow wave stimulation. So, it seems not enough to illustrate the seizure of epilepsy. But the H-H neuron model does not fit the real central neuron very well, especially at higher temperature. We will study further at other neuron models and look for parameters that bring a neuron to Hopf bifurcation at human body temperature.

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